
Stress around and at a Hole in an Infinite Plate

Abstract

This report details the classic analytical solution for the stress distribution around a circular hole in an infinite, isotropic, linearly elastic plate subjected to uniaxial tension. It outlines the foundational Kirsch equations, which describe the complete stress field. The focus is placed on the phenomenon of stress concentration at the hole's boundary, where the maximum stress is shown to be three times the remotely applied stress. The limitations of this ideal model and its extensions to more practical engineering scenarios are also discussed.

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1. Introduction

In mechanical and structural engineering, discontinuities such as holes, notches, or fillets are common design features. However, these features disrupt the smooth flow of stress through a component, creating localized areas of high stress. This phenomenon, known as **stress concentration**, is a critical factor in material failure, particularly under fatigue loading.

The problem of determining the stress field around a circular hole in a uniaxially loaded infinite plate is a fundamental case study in the theory of elasticity. Its analytical solution provides crucial insights into how stress concentrations arise and serves as a benchmark for analyzing more complex geometries.

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2. The Analytical Solution: Kirsch Equations

The exact solution for this problem was derived by Ernst Gustav Kirsch in 1898. For an infinite plate with a central circular hole of radius *a*, under a far-field uniaxial stress *σ*, the stress components in a polar coordinate system (*r*, *θ*) centered on the hole are given by:

- **Radial Stress (σrr):**

$$\sigma rr=2\sigma(1-r2a2)+2\sigma(1+3r4a4-4r2a2)\cos(2\theta)$$

- **Tangential (Hoop) Stress (σθθ):**

$$\sigma\theta\theta=2\sigma(1+r2a2)-2\sigma(1+3r4a4)\cos(2\theta)$$

- **Shear Stress ($\sigma r\theta$):**

$\sigma r\theta=-2\sigma(1-3r4a4+2r2a2)\sin(2\theta)$

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3. Stress at the Hole Boundary

The most critical stresses occur at the boundary of the hole itself (where *r = a*). At this free surface, the radial and shear stresses naturally become zero. The tangential (hoop) stress simplifies to:

$\sigma\theta\theta=\sigma(1-2\cos(2\theta))$

This equation reveals the distribution of stress around the circumference of the hole:

- Maximum Stress: At the points perpendicular to the applied load ($\theta=\pm90\circ$), the stress reaches its peak value:

$\sigma max=\sigma(1-2\cos(180\circ))=3\sigma$

- Minimum Stress: At the points parallel to the applied load ($\theta=0\circ$ and $180\circ$), the stress is compressive:

$\sigma min=\sigma(1-2\cos(0\circ))=-\sigma$

This gives rise to the theoretical **Stress Concentration Factor (Kt)**, defined as the ratio of the maximum local stress to the nominal remote stress. For this case, **Kt=3**. This means the presence of the hole triples the stress at two specific points, which are often the sites of crack initiation.

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4. Limitations and Practical Considerations

The Kirsch solution is based on several idealizations:

- The plate is **infinite**, meaning its boundaries are too far away to influence the stress field around the hole. For plates of **finite width**, the stress concentration is higher and requires correction factors.
- The material is perfectly **linearly elastic**, meaning it obeys Hooke's Law. In ductile materials, high-stress concentrations cause local yielding, which blunts the peak stress.
- The hole is **circular**. For other shapes, like ellipses, the stress concentration can be significantly higher, especially for sharp radii.

For complex geometries and material behaviors where analytical solutions are intractable, engineers use numerical methods like the **Finite Element Method (FEM)** to accurately predict stress distributions.

5. Conclusion

The analysis of stress around a circular hole demonstrates a foundational principle of mechanics: geometric discontinuities amplify stress. The theoretical stress concentration factor of 3 for a hole in an infinite plate under uniaxial tension is a cornerstone of mechanical design. This knowledge enables engineers to design components that can safely withstand their operational loads by accounting for these predictable stress peaks, thereby preventing premature failure.